

# **FPGA Based EKF Estimator for DTC Induction Motor Drives**

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- Mathematical model of IM
- Direct Torque Control (DTC) strategies of IM
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- Implementation of EKF algorithm in FPGA
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- **Induction Motors (IM) characteristics:**

- **Advantages:**

- 1) **Highly reliable, as its simple design has no brushes that could wear out,**
- 2) **No electrical sparking.**
- 3) **Available in single-phase and three-phase; three phase is an ideal choice for variable-speed applications.**
- 4) **Manufactured at a very low cost.**

- **Applications:**

- 1) **Traction and Propulsion systems**
- 2) **Manufacturing machines (CNC) used in variety of production lines such as Drilling, Grinding, Hardening and Etc..**

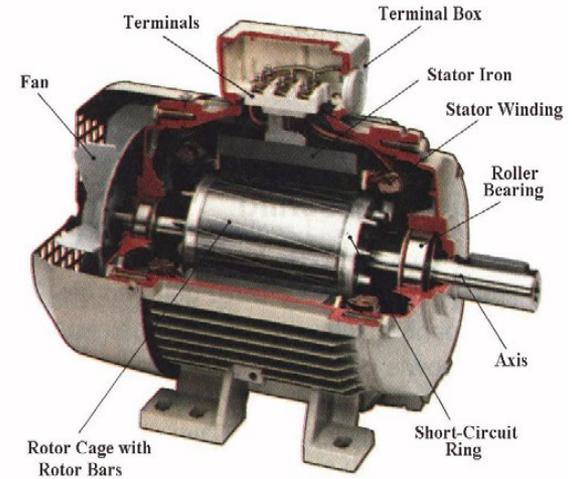
- **Drawback:**

- 1) **Less efficient and more sluggish than other motor types like DC Machines**

- **Solutions**

- **Use of effective speed controllers**

- 1) **Vector controller: Direct or Indirect Field Oriented (DFOC or IFOC)**
- 2) **Direct Torque Controller ( DTC)**



**Induction Motor Cross Sectional view**

# •Mathematical model of IM

## •Stator and Rotor voltage equations

$$\begin{cases} U_s^s = R_s I_s^s + \frac{d\psi_s^s}{dt} \\ 0 = R_r I_r^s - j\omega_r \psi_r^s + \frac{d\psi_r^s}{dt} \end{cases} \quad (1)$$

## •Stator and Rotor flux equations

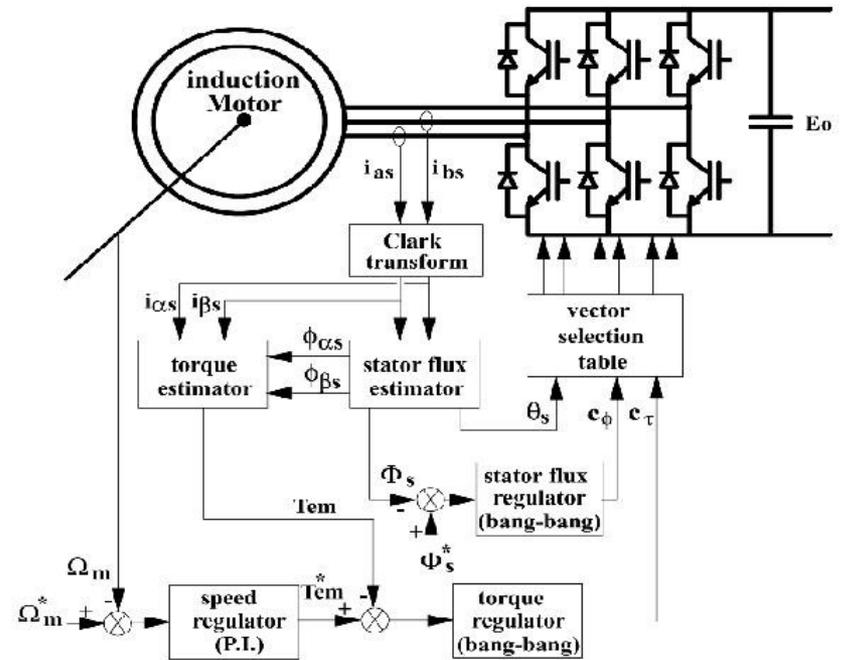
$$\begin{cases} \psi_s^s = L_s I_s^s + L_m I_r^s \\ \psi_r^s = L_r I_r^s + L_m I_s^s \end{cases} \quad (2)$$

## •Mechanical equation

$$J \frac{d\omega_r}{dt} + f_v \omega_r = T_e - T_l \quad (3)$$

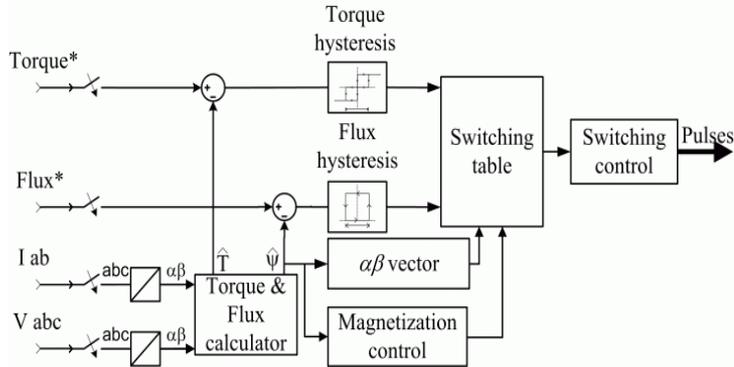
## •Electromagnetic Torque equation

$$T_e = \frac{3p}{2} \frac{L_m}{L_s L_r - L_m^2} \psi_s \otimes \psi_r = \psi_s \psi_r \sin \theta_{sr} \quad (4)$$

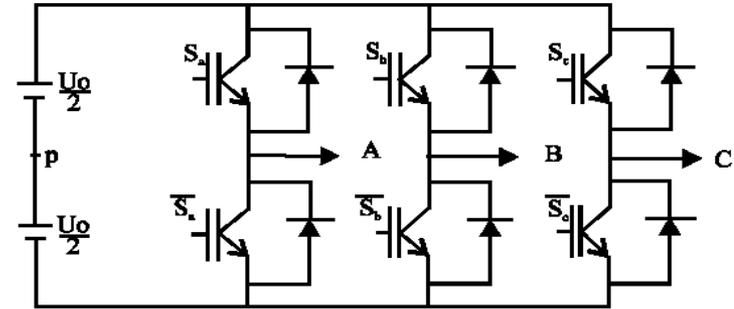


DTC black diagram using speed sensor

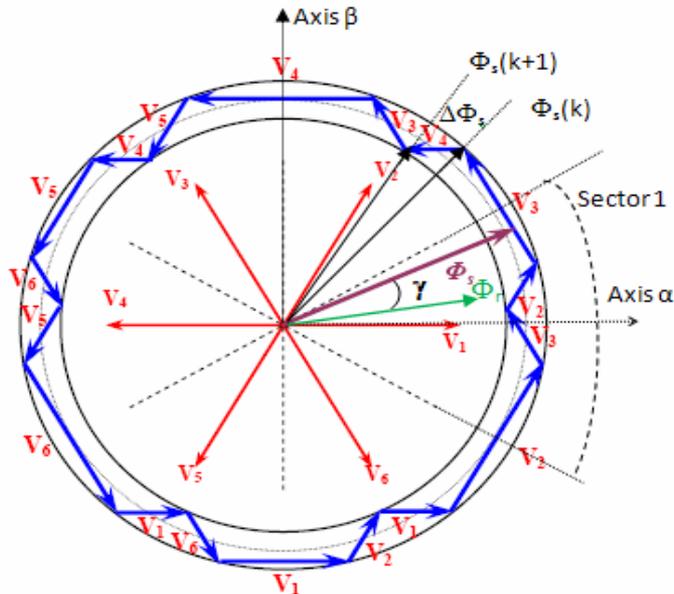
# •Direct Torque Control (DTC) strategies of IM



Flux and Torque hysteresis bands



Three phase PWM voltage source inverter



Sector and voltage vectors

$$\Delta\varphi_s \approx U_s \Delta t \quad (5)$$

		Sectors ( $S_i; i=1-6$ )					
$\tau_f$	$\tau_T$	$S_1$	$S_2$	$S_3$	$S_4$	$S_5$	$S_6$
1	1	$V_2$	$V_3$	$V_4$	$V_5$	$V_6$	$V_1$
	0	$V_7$	$V_0$	$V_7$	$V_0$	$V_7$	$V_0$
	-1	$V_6$	$V_1$	$V_2$	$V_3$	$V_4$	$V_5$
0	1	$V_3$	$V_4$	$V_5$	$V_6$	$V_1$	$V_2$
	0	$V_0$	$V_7$	$V_0$	$V_7$	$V_0$	$V_7$
	-1	$V_5$	$V_6$	$V_1$	$V_2$	$V_3$	$V_4$

Switching lookup table

$$\begin{aligned}
 V_1 &= 001 & V_2 &= 010 & V_3 &= 011 \\
 V_4 &= 100 & V_5 &= 101 & V_6 &= 101 \\
 V_0 &= 000 = V_7 = 111
 \end{aligned}$$

**1: Connected to positive source**  
**0: Connected to negative source**

## •State space model of IM

$$\begin{cases} \dot{x} = f(x, u) + w \\ y = Cx + v \end{cases} \quad (6)$$

$$x = [i_{s\alpha}, i_{s\beta}, \varphi_{r\alpha}, \varphi_{r\beta}]$$

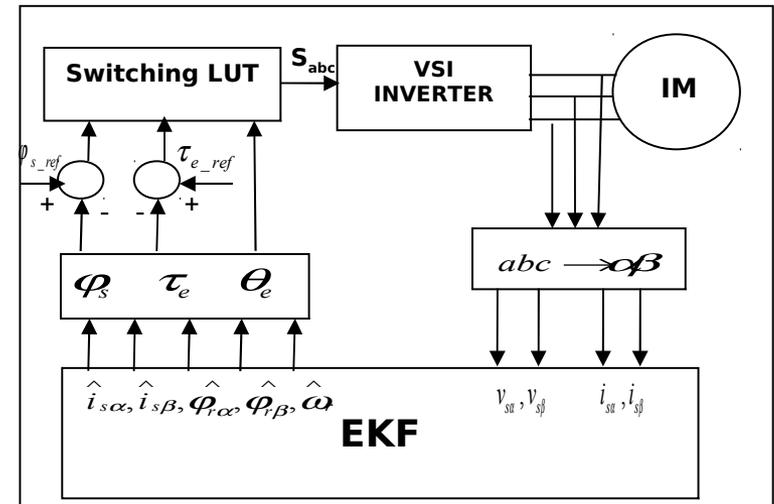
$$U = [V_{s\alpha}, V_{s\beta}]^T, y = [i_{s\alpha}, i_{s\beta}]^T$$

**W:** System disturbance, zero-mean Gaussian noises

**V:** Measurement disturbance, zero-mean Gaussian noises

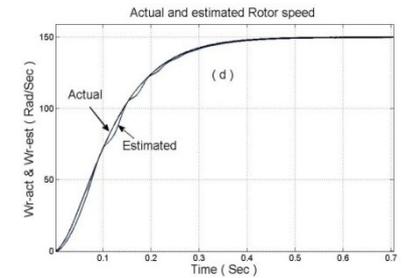
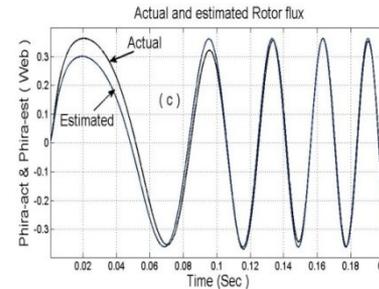
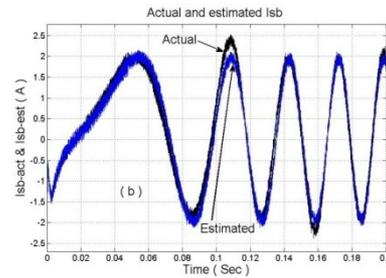
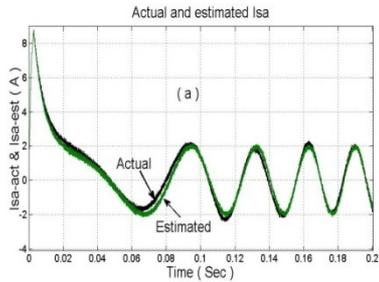
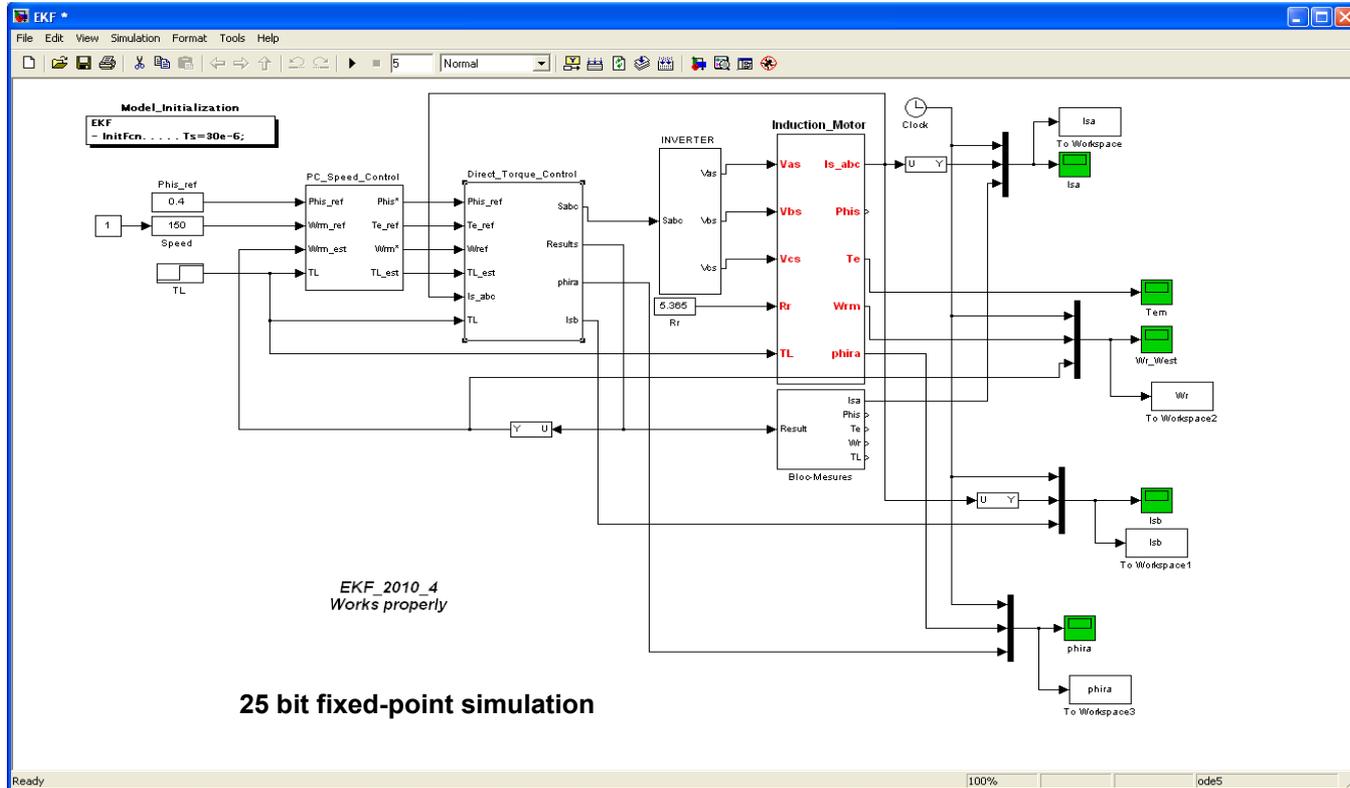
$$f(x, u) = \begin{bmatrix} (1 - T_s \gamma) i_{s\alpha} + T_s \frac{MR_r}{L_r^2 k} \varphi_{r\alpha} + T_s \frac{M\omega_r}{L_r k} \varphi_{r\beta} + T_s \frac{1}{k} V_{s\alpha} \\ (1 - T_s \gamma) i_{s\beta} - T_s \frac{M\omega_r}{L_r k} \varphi_{r\alpha} + T_s \frac{MR_r}{L_r^2 k} \varphi_{r\beta} + T_s \frac{1}{k} V_{s\beta} \\ T_s \frac{M}{T_r} i_{s\alpha} + (1 - \frac{T_s}{T_r}) \varphi_{r\alpha} - T_s \omega_r \varphi_{r\beta} \\ T_s \frac{M}{T_r} i_{s\beta} + T_s \omega_r \varphi_{r\alpha} + (1 - \frac{T_s}{T_r}) \varphi_{r\beta} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, k = \frac{M}{\sigma L_s L_r}, \gamma = \frac{R_s}{\sigma L_s} + \frac{R_r M^2}{L_r^2 \sigma L_s}$$



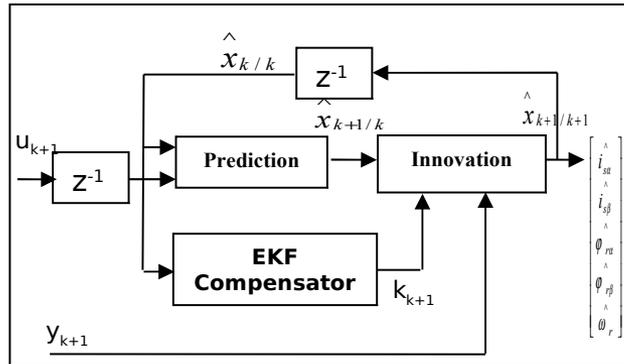
Block diagram of EKF- based DTC control system

# •DTC Simulation in MATLAB SIMULINK

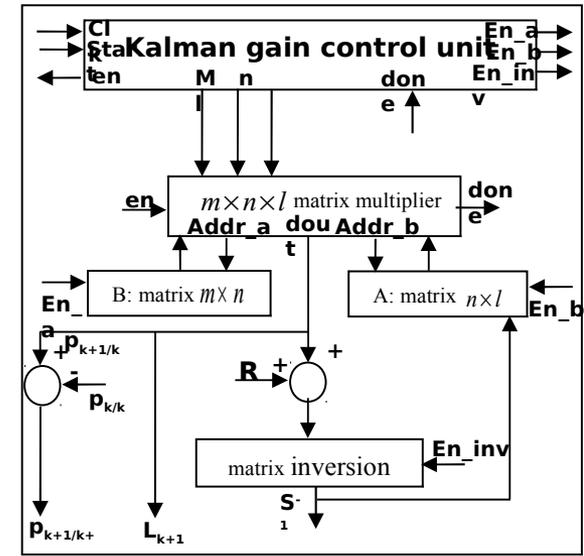


Actual and estimated state space variables: a)  $I_{sa}$ , b)  $I_{sb}$ , c)  $\Phi_{ra}$ , d)  $\omega_r$

# Implementation of EKF algorithm in FPGA



Systolic of the EKF algorithm



FPGA-based architecture of the EKF compensator

## 1) Prediction state vector:

$$\hat{x}(k+1/k) = F(\hat{x}(k/k), U(k)) \quad (7), \text{Where: } F(k) = \frac{\partial f}{\partial x} \Big|_{x(k) = \hat{x}(k/k)}$$

## 2) Prediction covariance computation:

$$p(k+1/k) = F(k)p(k/k)F(k)^T + Q \quad (8)$$

Q: System covariance matrix characterizing W  
p: State vector covariance matrix

## 3) Kalman gain computation:

$$L(k+1) = p(k+1/k)C(k)^T (C(k)P(k+1)C(k)^T + R)^{-1} \quad (9)$$

R: Measurement covariance matrix characterizing V

## 4) State vector estimation or innovation step

$$\hat{x}(k+1/k+1) = \hat{x}(k+1/k) + L(k+1)(y(k+1) - C\hat{x}(k+1/k)) \quad (10)$$

# •Implementation of EKF algorithm in FPGA

## Fixed point math packages :

### Features:

- 1) It is a step between integer and floating point math
- 2) It is almost as fast as numeric\_std arithmetic capable to represent fractional numbers
- 3) A fixed-point number has an assigned width and an assigned location for the decimal point so as the integer and fractional parts are recognizable
- 4) Because it is based on integer math, it is extremely efficient, as long as the data does not vary too much in magnitude

## •Fixed point number representation and arithmetic operations:

$y = 6.5 = "00110.10000"$ , 10 bits unsigned Fixed point representation of number 6.5

$y = 6.5 = "000110.11000"$ , 11 bits signed Fixed point representation of number 6.75

Format in VHDL:

```
signal a, : ufixed (4 downto -5);
```

```
signal b, : sfixed (5 downto -5);
```

The most often used conversions and arithmetic operations :

```
a<= to_sfixed(6.5, a'high, a'low);
```

```
b<= to_sfixed(3.15, b'high,b'low);
```

```
m<= to_slv(a);
```

```
c<=(a+b, Max(a'high, b'high)+1 downto Min(a'low, b'low));
```

```
d<=(a-b , Max(a'high, b'high)+1 downto Min(a'low, b'low));
```

```
e<=(a*b , a'high+b'high+1 downto a'low+b'low);
```

```
acc<=resize(e, acc'high,acc'low);
```

### Usage model:

```
library ieee, ieee_proposed;
use ieee.std_logic_1164.all;
use ieee.numeric_std.all;
use ieee_proposed.fixed_float_types.all;
use ieee_proposed.fixed_pkg.all;
```

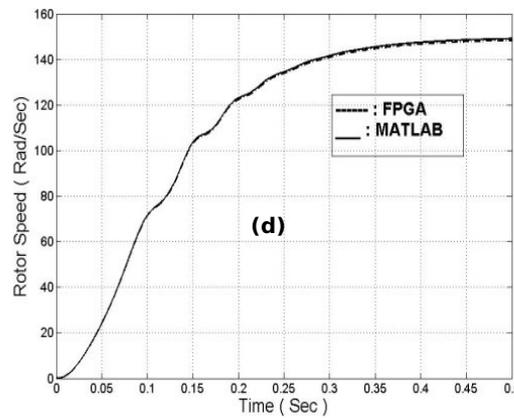
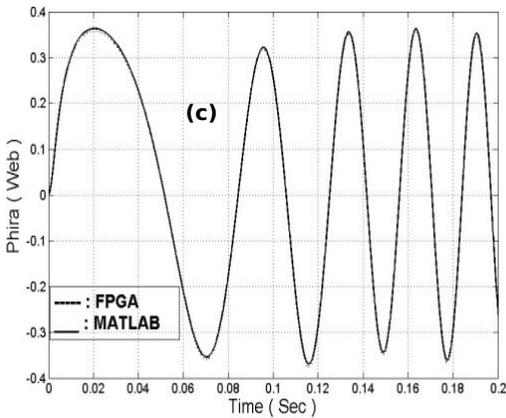
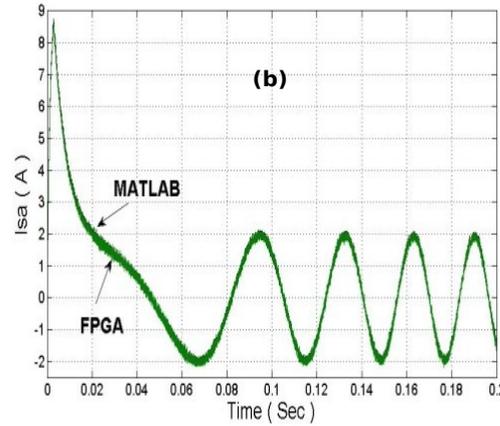
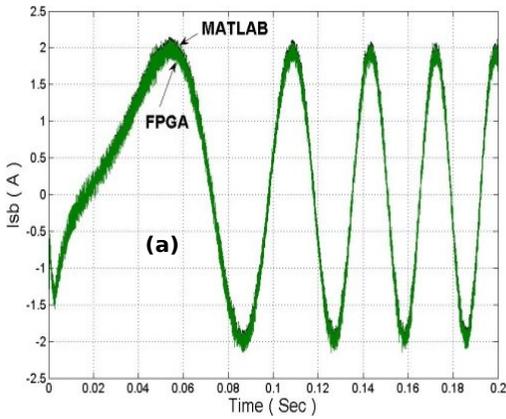
```
entity matrix is
  Port ( clk : in STD_LOGIC;
        start : in STD_LOGIC;
        Go : out STD_LOGIC);
end matrix;
```

```
architecture arc_mat of matrix is
  component mult is
    port ( clk : in STD_LOGIC;
          en : in STD_LOGIC;
          dir : std_logic;
          Done : out STD_LOGIC;
          Dout : out std_logic_vector (43 downto 0);
          kk0 : out std_logic_vector (2 downto 0);
          jj0 : out std_logic_vector (2 downto 0);
          ii0 : out std_logic_vector (2 downto 0);
          ii4 : out std_logic_vector (2 downto 0);
          jj4 : out std_logic_vector (2 downto 0);
          Din1 : in std_logic_vector (24 downto 0);
          Din2 : in std_logic_vector (24 downto 0);
          mm : in std_logic_vector(2 downto 0);
          nn : in std_logic_vector (2 downto 0);
          ll : in std_logic_vector (2 downto 0);
          end component;
```

```
-----
subtype sfixed25 is sfixed( 10 downto -14);
subtype sfixed26 is sfixed( 10 downto -15);
subtype sfixed18 is sfixed( 10 downto -7);
```

•  
•  
•

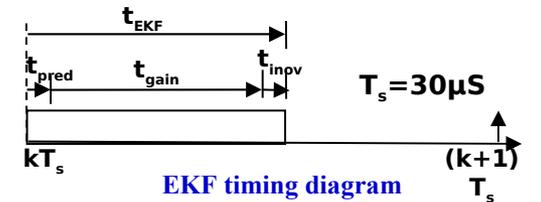
# •Implementation of EKF algorithm in FPGA



HIL results, a)  $I_{sa}$ , b)  $I_{sb}$ , c)  $Phira$ , d)  $W_r$



Hardware in Loop



$$t_{EKF} = 18.68\mu S \quad t_{pred} = 1.1\mu S \quad t_{inov} = 1.61\mu S$$

Table 1, FPGA resources usage

	Available	Used	Utilization
slice registers	32640	41	1%
Slice LUTs	32640	195	1%
Logics	32640	195	1%
25×18 Hw multipliers	136	9	7%



## •Conclusions

- The design and implementation of the FPGA-based Extended Kalman Filter ( EKF) estimator was presented.
- The implemented EKF algorithm as a part of DTC control system was described focusing on the FPGA architecture.
- Simulation and hardware in loop (HIL) results were provided in order to validate the performance and effectiveness of the developed design and the time/area analysis was then presented.
- An examination of time/ area performance of FPGA architecture reveals that the whole DTC controller is implementable using one single stand alone Xilinx ML506 FPGA Platform.
- The total recourses used for implementing EKF algorithm is only 10% of available FPGA hardware.  
The rest of the resource is quite enough to implement all low speed adders or multipliers required  
for designing the remaining parts of complete DTC controller.

***Thank you***