$*\mathcal{X}$

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Quelques mots clés

- Les fils.
- La reconfiguration des architectures.

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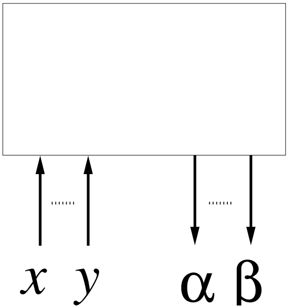
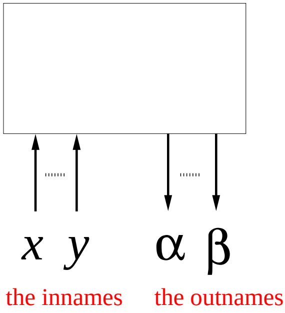
- La reconfiguration des architectures.

- Mine à logiques.

A net

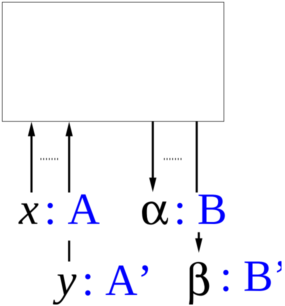


A net



A net

Connections are typed.



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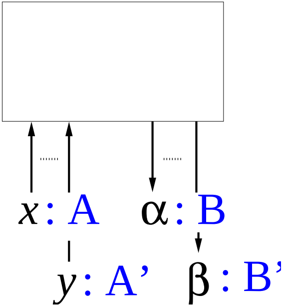
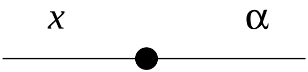
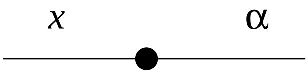


The basic net



The outname α emits what it receives from the inname x.

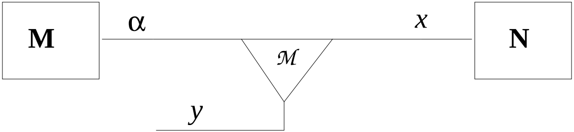
The basic net



The outname α emits what it receives from the inname x.

This net has an in port x and out port α.

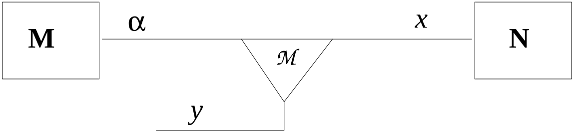
The mediator



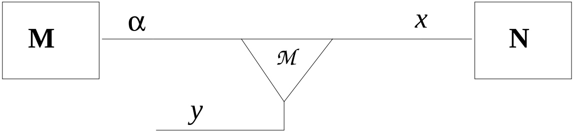
M et N are connected through

- the inname x for N
- and the outname α for M.

The mediator

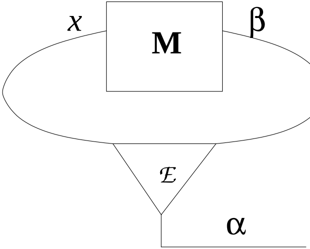


The mediator



The mediator y is provided through a specific inname y.

The exporter



M takes something in *type*(x) and yields something in *type*(β).

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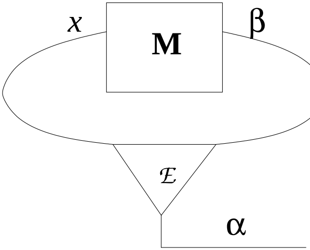


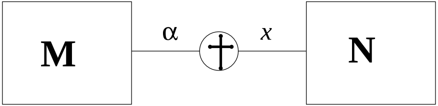
M takes something in $\textit{type}(x)$ and yields something in $\textit{type}(\beta)$.

It exports something in $\textit{type}(x) \to \textit{type}(\beta)$, on α.

The weld or cut

It is the most elementary operation, which «welds» two nets

- through their outname (for left net) and
- through their inname (for the right net).



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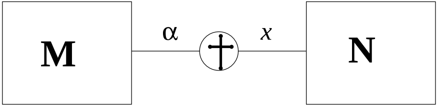

The outname α of M is «welded» to the inname x of N.

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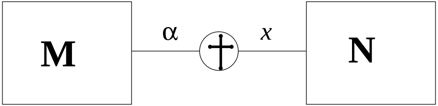
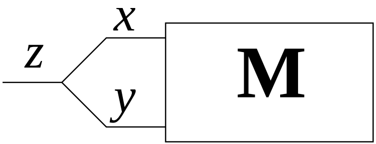


The outname α of M is «welded» to the inname x of N.

As we are going to be interested to the inverse operation,
we also speak about cut.

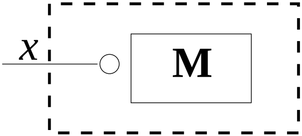
The contraction



The weakening

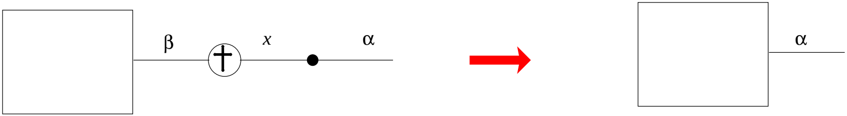
Since terms are linear. Names must occurs.

What do we do when one does not want a name to occurs ?

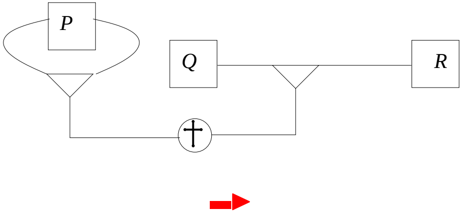


cap-cap

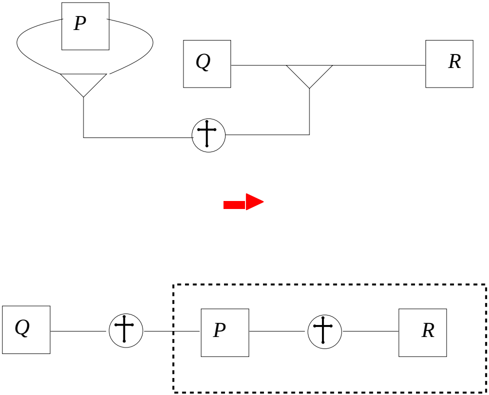
More generally



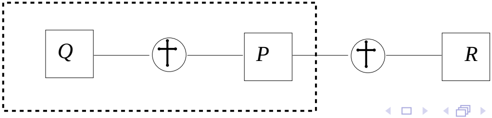
exp-med



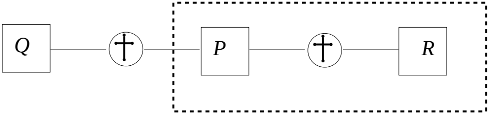
exp-med

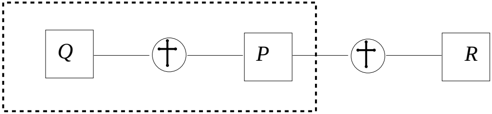


or



Actually, by the fact the same term can reduce to two terms





we get a non confluent system.

Binding names

Unlike most of the languages with bounded variables :
lambda-calculus, quantification, fixed points, etc.

mediators, exports and welds bind **two** names :
an outname and an inname.

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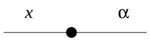
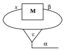
Bounded names wear hats :

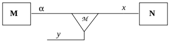
For instance :

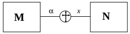
$$M_1 \widehat{\alpha} \oplus \widehat{x} M_2$$

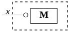
where $\oplus$ is a binary operator.

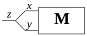

x
α

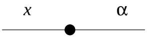

x
M
β
α


M
α
ℳ
x
N
y

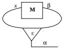

M
α
x
N


x
M

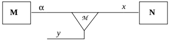

z
x
y
M



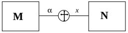
$$\langle x.\alpha\rangle$$



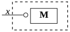
$$\widehat{x}\, M\, \widehat{\beta}\cdot\alpha$$



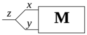
$$P\widehat{\beta}\ [x]\ \widehat{z}Q$$



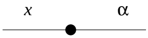
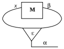
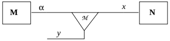
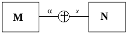
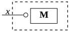
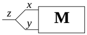
$$P\widehat{\alpha}\ \dagger\ \widehat{x}Q$$

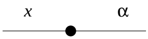


$$x\circ\{M\}$$

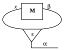


$$z<^{\widehat{x}}_{\widehat{y}}\langle M]$$

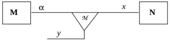
	$\langle x.\alpha\rangle$	capsule
	$\widehat{x}\, M\, \widehat{\beta}\cdot\alpha$	export
	$P\widehat{\beta}\,[x]\,\widehat{z}Q$	export
	$P\widehat{\alpha} \dagger \widehat{x}Q$	cut
	$x \circ \{M\}$	weakening
	$z{<}^{\widehat{x}}_{\widehat{y}}\langle M]$	contraction



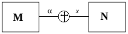
$\langle x.\alpha \rangle$ — capsule



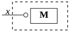
$\widehat{x}\, M\, \widehat{\beta} \cdot \alpha$ — export



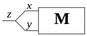
$P \widehat{\beta}\ [x]\ \widehat{z} Q$ — export



$P \widehat{\alpha}\ \dagger\ \widehat{x} Q$ — obèle



$x \circ \{M\}$ — weakening



$z {<}^{\widehat{x}}_{\widehat{y}} \langle M]$ — contraction

Terms are linear

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This means that each bound name accurs once and only once.

The same for the free names.

Logical rules

$$
\begin{array}{llll}
(exp-med) & : & (\widehat{y}\,P\,\widehat{\beta}\cdot\alpha)\widehat{\alpha} \;\dagger\; \widehat{x}(Q\widehat{\gamma}\;[x]\;\widehat{z}R) & \rightarrow \begin{cases} (Q\widehat{\gamma} \;\dagger\; \widehat{y}P)\widehat{\beta} \;\dagger\; \widehat{z}R \\ Q\widehat{\gamma} \;\dagger\; \widehat{y}(P\widehat{\beta} \;\dagger\; \widehat{z}R) \end{cases} \\
(cap-ren) & : & \langle y.\alpha\rangle\widehat{\alpha} \;\dagger\; \widehat{x}\langle x.\beta\rangle & \rightarrow \langle y.\beta\rangle \\
(exp-ren) & : & (\widehat{y}\,P\,\widehat{\beta}\cdot\alpha)\widehat{\alpha} \;\dagger\; \widehat{x}\langle x.\gamma\rangle & \rightarrow \widehat{y}\,P\,\widehat{\beta}\cdot\gamma \\
(med-ren) & : & \langle y.\alpha\rangle\widehat{\alpha} \;\dagger\; \widehat{x}(P\widehat{\beta}\;[x]\;\widehat{z}Q) & \rightarrow P\widehat{\beta}\;[y]\;\widehat{z}Q
\end{array}
$$

The implicative sequent calculus (the rules)

$$\frac{}{\Gamma, A \vdash \Delta, A}\ (ax)$$

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$$\frac{\Gamma \vdash A, \Delta \qquad \Gamma, B \vdash \Delta}{\Gamma, A \rightarrow B \vdash \Delta}\ (\rightarrow L) \qquad \frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \rightarrow B, \Delta}\ (\rightarrow R)$$

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$$\frac{\Gamma \vdash A, \Delta \qquad \Gamma, A \vdash \Delta}{\Gamma \vdash \Delta}\ (cut)$$

The implicative sequent calculus (the rules)

$$\frac{}{\Gamma, A \vdash \Delta, A} \; (ax)$$

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$$\frac{\Gamma \vdash A, \Delta \qquad \Gamma, A \vdash \Delta}{\Gamma \vdash \Delta} \; (cut)$$

$$\frac{\Gamma \vdash \Delta}{\Gamma, A \vdash \Delta} \; (L - Weak) \qquad \frac{\Gamma \vdash \Delta}{\Gamma \vdash A, \Delta} \; (R - Weak)$$

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$$\frac{\Gamma \vdash \Delta}{\Gamma, A \vdash \Delta}\ (L - Weak) \qquad \frac{\Gamma \vdash \Delta}{\Gamma \vdash A, \Delta}\ (R - Weak)$$

$$\frac{\Gamma, A, A \vdash \Delta}{\Gamma, A \vdash \Delta}\ (L - Contr) \qquad \frac{\Gamma \vdash A, A, \Delta}{\Gamma \vdash A, \Delta}\ (R - Contr)$$

Typing $*\mathcal{X}$

$$\frac{}{\langle x.\alpha \rangle : \Gamma, x : A \vdash \Delta, \alpha : A} \; (cap)$$

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$$\frac{M : \Gamma \vdash \alpha : A, \Delta \qquad N : \Gamma, x : B \vdash \Delta}{M\widehat{\alpha}\,[y]\,\widehat{x}N : \Gamma, A \to B \vdash \Delta} \; (med) \qquad \frac{M : \Gamma, x : A \vdash \alpha : B, \Delta}{\widehat{x}\,M\,\widehat{\alpha} \cdot \beta : \Gamma \vdash \beta : A \to B, \Delta} \; (exp)$$

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$$\frac{P : \Gamma \vdash \alpha : A, \Delta \qquad Q : \Gamma, x : A \vdash \Delta}{P\widehat{\alpha}\ \dagger\ \widehat{x}Q : \Gamma \vdash \Delta}\ (cut)$$

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$$\frac{M : \Gamma \vdash \Delta}{x \circ \{M\} : \Gamma, x : A \vdash \Delta}\ (L - Weak) \qquad \frac{M : \Gamma \vdash \Delta}{\{M\} \circ \alpha : \Gamma \vdash \alpha : A, \Delta}\ (R - Weak)$$

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$$\frac{P : \Gamma \vdash \alpha : A, \Delta \qquad Q : \Gamma, x : A \vdash \Delta}{P\widehat{\alpha}\ \dagger\ \widehat{x}Q : \Gamma \vdash \Delta}\ (cut)$$

$$\frac{M : \Gamma \vdash \Delta}{x \circ \{M\} : \Gamma, x : A \vdash \Delta}\ (L-Weak) \qquad \frac{M : \Gamma \vdash \Delta}{\{M\} \circ \alpha : \Gamma \vdash \alpha : A, \Delta}\ (R-Weak)$$

$$\frac{M : \Gamma, x : A, y : A \vdash \Delta}{z {<}^{\widehat{x}}_{\widehat{y}} \langle M] : \Gamma, z : A \vdash \Delta}\ (L-Contr) \qquad \frac{M : \Gamma \vdash \alpha : A, \beta : A, \Delta}{[M\rangle^{\widehat{\alpha}}_{\widehat{\beta}}{>}\gamma : \Gamma \vdash \gamma : A, \Delta}\ (R-Contr)$$

Curry-Howard-de Bruijn correspondence

Curry-Howard-de Bruijn correspondence

Curry-Howard-de Bruijn correspondence

Typing rules of $*\mathcal{X}$ are implicative sequent rules.

Net reductions correspond to cut elimination.

Curry-Howard-de Bruijn correspondence



Typing rules of $*\mathcal{X}$ are implicative sequent rules.

Net reductions correspond to cut elimination.

We did not give all the rules for cut elimination.

How about the case when logical rules are not applicable ?

When M_1 is neither $\widehat{y}\, P\, \widehat{\beta} \cdot \alpha$ nor $\langle y.\alpha \rangle$
or M_2 is neither $P\widehat{\beta}\ [x]\ \widehat{y}Q$ nor $\langle x.\beta \rangle$

in $M_1 \widehat{\alpha}\ \dagger\ \widehat{x} M_2$

one "distributes" the "cut" in order to "eliminate" it.

Bending the dagger or activating the cuts

When x or α is not introduced, one bends the †'s to show in which direction (left $\nearrow$ or right $\nwarrow$),
the x†'s have to be distributed.

Bending the dagger or activating the cuts

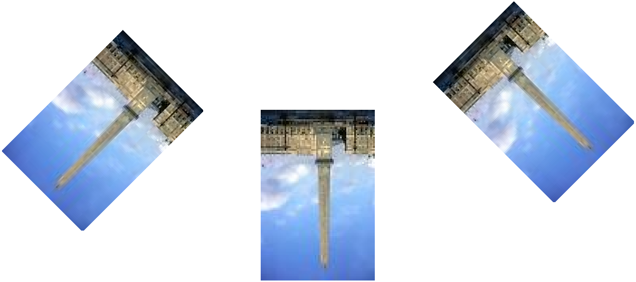
When x or α is not introduced, one bends the †'s to show in which direction (left ⸸̸ or right ⸸̷),
the x†'s have to be distributed.

Bending the dagger or activating the cuts

When x or α is not introduced, one bends the $\dagger$'s to show in which direction (left $\dagger$ or right $\dagger$),
the $x\dagger$'s have to be distributed.

Bending the dagger or activating the cuts

When x or α is not introduced, one bends the †'s to show in which direction (left $\dagger$ or right $\dagger$),
the x†'s have to be distributed.



Bending the dagger or activating the cuts

When x or α is not introduced, one bends the †'s to show in which direction (left $\nearrow\!\!\!\!\dagger$ or right $\nwarrow\!\!\!\!\dagger$),
the †'s have to be distributed.

Bending the dagger

$$(act\text{-L}) \quad P\widehat{\alpha} \dagger \widehat{x}Q \quad \longrightarrow \quad P\widehat{\alpha} \nearrow\!\!\!\!\dagger \widehat{x}Q$$

$$\text{with } P \neq \widehat{y}\, P' \, \widehat{\beta} \cdot \alpha \text{ and } P \neq \langle y.\alpha \rangle$$

$$(act\text{-R}) \quad P\widehat{\alpha} \dagger \widehat{x}Q \quad \longrightarrow \quad P\widehat{\alpha} \nwarrow\!\!\!\!\dagger \widehat{x}Q$$

$$\text{with } Q \neq Q'\widehat{\beta}\, [x]\, \widehat{y}Q'' \text{ and } Q \neq \langle x.\beta \rangle$$

Bending the dagger or activating the cuts

When x or α is not introduced, one bends the †'s to show in which direction (left $\nearrow\!\!\!\!\dagger$ or right $\nwarrow\!\!\!\!\dagger$),
the †'s have to be distributed.

Bending the dagger

$$(act\text{-L}) \quad P\widehat{\alpha} \dagger \widehat{x}Q \quad \to \quad P\widehat{\alpha} \nearrow\!\!\!\!\dagger\ \widehat{x}Q$$
$$\text{with } P \neq \widehat{y}\,P'\,\widehat{\beta}\cdot\alpha \text{ and } P \neq \langle y.\alpha\rangle$$

$$(act\text{-R}) \quad P\widehat{\alpha} \dagger \widehat{x}Q \quad \to \quad P\widehat{\alpha} \nwarrow\!\!\!\!\dagger\ \widehat{x}Q$$
$$\text{with } Q \neq Q'\widehat{\beta}\,[x]\,\widehat{y}Q'' \text{ and } Q \neq \langle x.\beta\rangle$$

Terms in which †'s are not bended are called pure.

Left actions

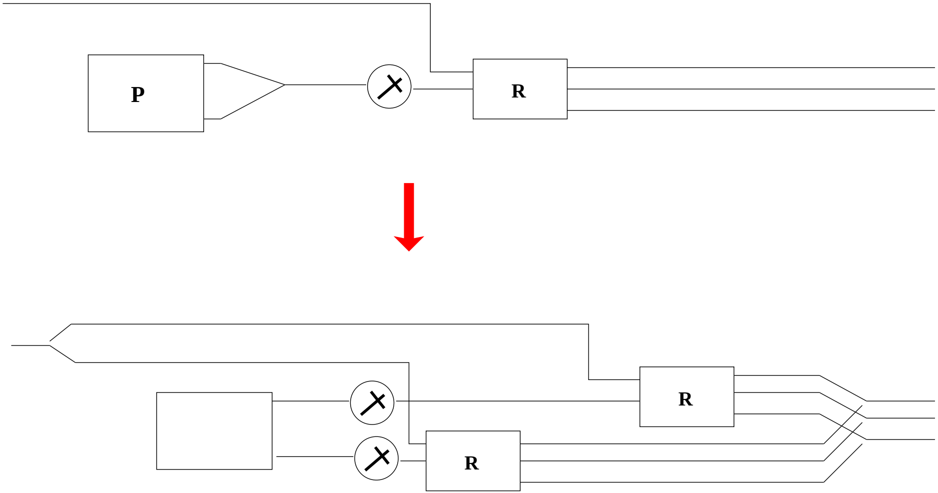
$$(\mathit{deact}): \quad \langle y.\alpha\rangle\widehat{\alpha} \not\dagger \widehat{x}P \;\Rightarrow\; \langle y.\alpha\rangle\widehat{\alpha} \dagger \widehat{x}P$$

$$(\mathit{med}-\mathit{deact}): \quad (\widehat{y}\, Q\, \widehat{\beta}\cdot\alpha)\widehat{\alpha} \not\dagger \widehat{x}P \;\Rightarrow\; (\widehat{y}\, Q\, \widehat{\beta}\cdot\alpha)\widehat{\alpha} \dagger \widehat{x}P$$

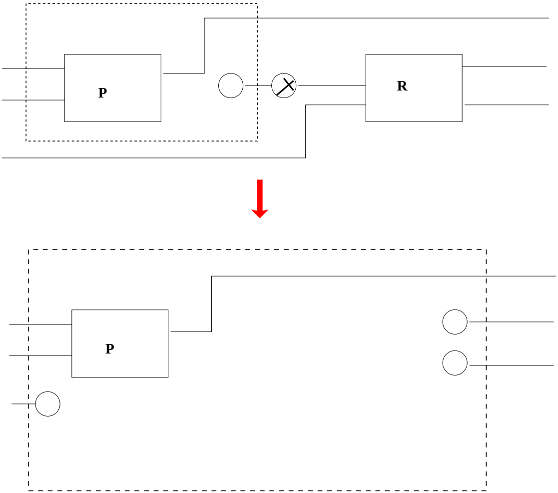
$(\mathit{cont}-\mathit{dupl})$: Through a picture

$(\mathit{weak}-\mathit{cancel})$: Through a picture

Rule cont-dupl



Rule weak-cancel



Left propagation

$$(\mathrm{L1}): \quad \langle y.\beta\rangle\widehat{\alpha} \not\dagger \widehat{x}P \;\Rightarrow\; \langle y.\beta\rangle, \qquad \beta \neq \alpha$$

$$(\mathrm{L3}): \quad (\widehat{y}\, Q\, \widehat{\beta} \cdot \gamma)\widehat{\alpha} \not\dagger \widehat{x}P \;\Rightarrow\; \widehat{y}\,(Q\widehat{\alpha} \not\dagger \widehat{x}P)\,\widehat{\beta} \cdot \gamma, \qquad \gamma \neq \alpha$$

$$(\mathrm{L4}): \quad (Q\widehat{\beta}\,[z]\,\widehat{y}R)\widehat{\alpha} \not\dagger \widehat{x}P \;\Rightarrow\; (Q\widehat{\alpha} \not\dagger \widehat{x}P)\widehat{\beta}\,[z]\,\widehat{y}R, \qquad x \in Q$$

$$(\mathrm{L5}): \quad (Q\widehat{\beta} \dagger \widehat{y}R)\widehat{\alpha} \not\dagger \widehat{x}P \;\Rightarrow\; (Q\widehat{\alpha} \not\dagger \widehat{x}P)\widehat{\beta} \dagger \widehat{y}R, \qquad x \in Q$$

⋮

Renaming

1. $P\widehat{\delta} \dagger \widehat{z}\langle z.\alpha\rangle \Rightarrow P[\alpha/\delta]$, if P is pure.
2. $\langle z.\alpha\rangle\widehat{\alpha} \dagger \widehat{x}P \Rightarrow P[z/x]$, if P is pure.

A question

How about describing real nets ?